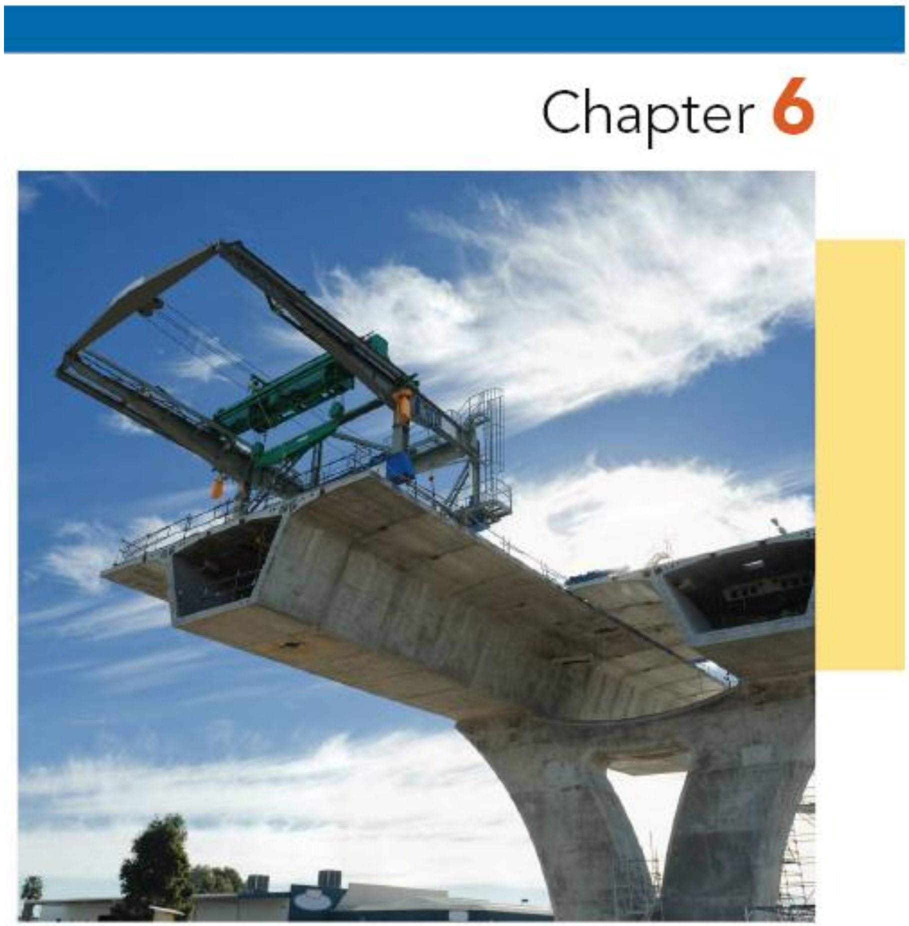
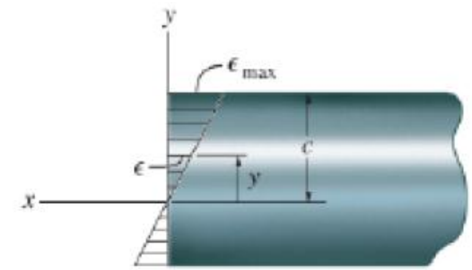
BENDING STRESS



The deck of this bridge has been designed on the basis of its ability to resist bending stress.

6.4 The Flexure Formula

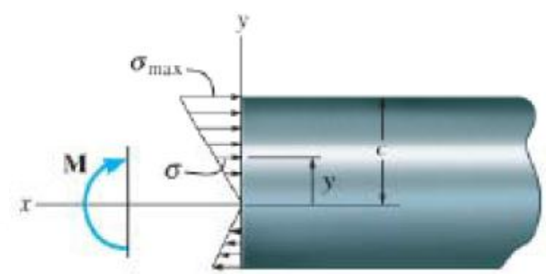
- Assume that material behaves in a linear-elastic manner so that Hooke's law applies.
- A *linear variation of normal strain must* then be the consequence of a *linear variation in normal stress*
- Applying Hooke's law to Eqn 6-8,



Normal strain variation (profile view)

(a)

Equation 6-9 $$\sigma = -(y/c)\sigma_{max}$$

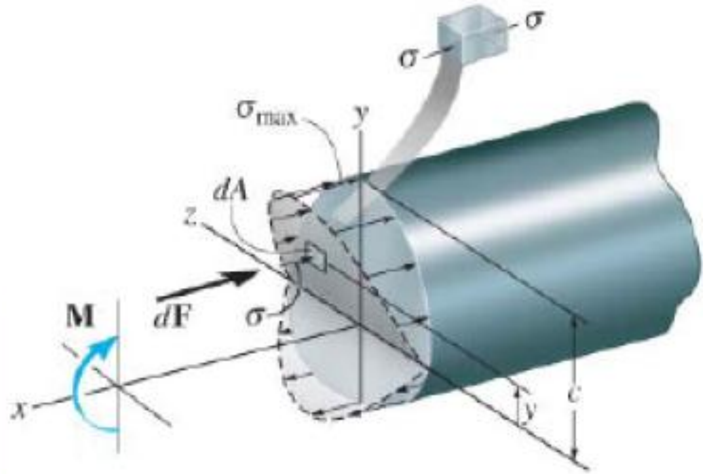


Bending stress variation (profile view)

(b)

BENDING STRESS

- By mathematical expression, equilibrium equations of moment and forces, we get



Bending stress variation

(c)

Equation 6-10 $$\int_A y\, dA = 0$$

Equation 6-11 $$M = \frac{\sigma_{max}}{c} \int_A y^2\, dA$$

- The integral represents the *moment of inertia* of x-sectional area, computed about the neutral axis. We symbolize its value as *I*.

BENDING STRESS

- Hence, Eqn 6-11 can be solved and written as

Equation 6-12 $$\sigma_{max} = \frac{Mc}{I}$$

$\sigma_{\max}$ = maximum normal stress in member, at a pt on x-sectional area farthest away from neutral axis

M = resultant internal moment, computed about neutral axis of x-section

I = moment of inertia of x-sectional area computed about neutral axis

c = perpendicular distance from neutral axis to a pt farthest away from neutral axis, where $\sigma_{\max}$ acts

BENDING STRESS

- Normal stress at intermediate distance y can be determined from

Equation 6-13 $$\sigma = -\frac{My}{I}$$

- σ is -ve as it acts in the -ve direction (compression)
- Equations 6-12 and 6-13 are often referred to as the *flexure formula*.

Important Points

- ✓ The cross section of a straight beam ***remains plane*** when the beam deforms due to bending. This causes ***tensile stress*** on one portion of the cross section and ***compressive stress*** on the other portion. In between these portions, there exists the ***neutral axis*** which is subjected to ***zero stress***.

- ✓ • Due to the deformation**, the *longitudinal strain*** varies *linearly* from **zero** at the neutral axis **to a maximum** at the outer fibers of the beam. Provided the material is homogeneous and linear elastic, then the *stress* also varies in a *linear* fashion over the cross section.

Important Points

- ✓ The neutral axis passes through the *centroid* of the cross-sectional area. This result is based on the fact that the resultant normal force acting on the cross section must be zero.

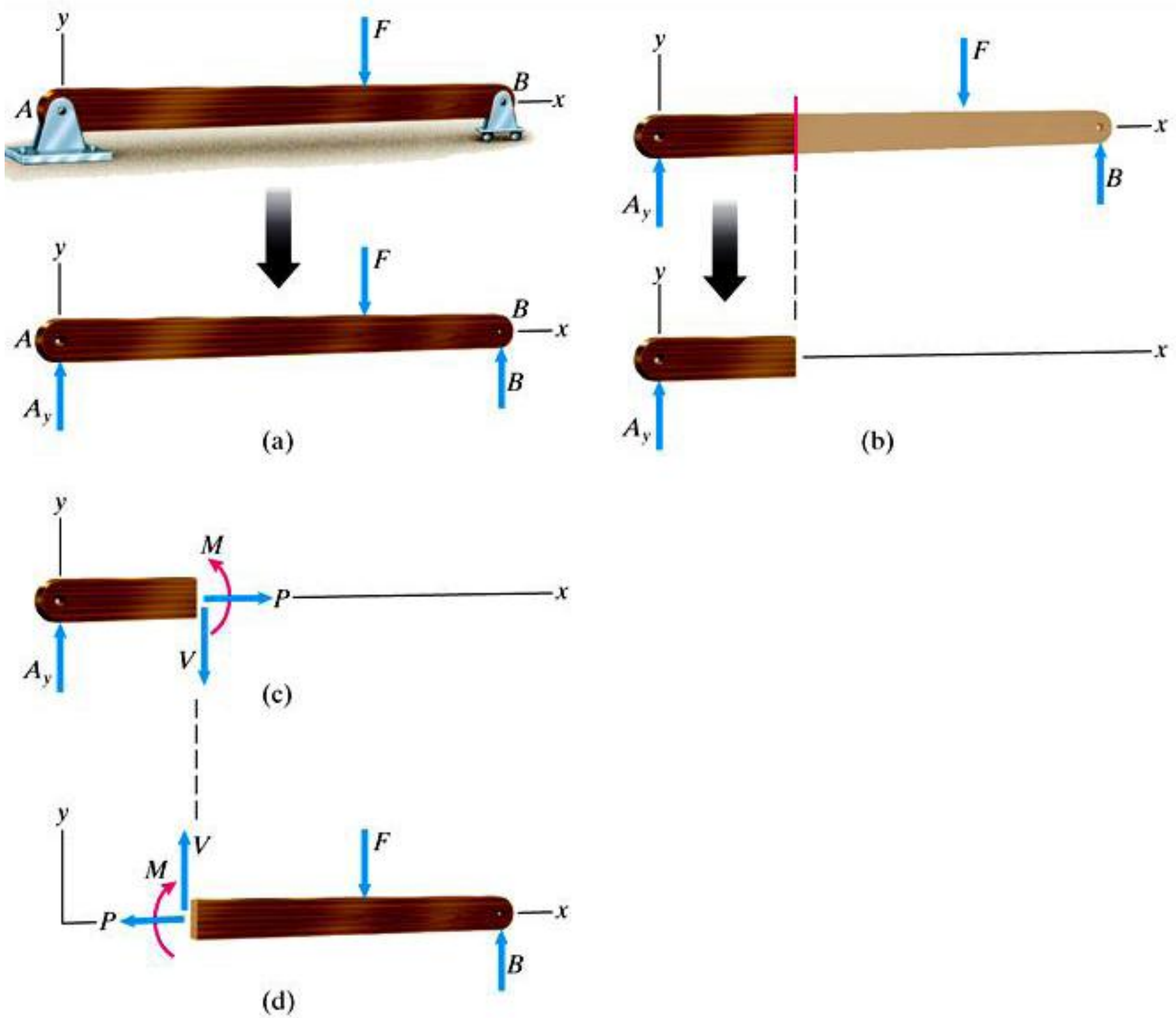
- ✓ The flexure formula is based on the requirement that the resultant internal moment on the cross section is equal to the moment produced by the normal stress distribution about the neutral axis.

Procedure for Analysis

❖ Internal Moment.

- Section member at point where bending or normal stress is to be determined and obtain internal moment M at the section.
- Centroidal or neutral axis for cross-section must be known since M is computed about this axis.
- If absolute maximum bending stress is to be determined, then draw moment diagram in order to determine the maximum moment in the diagram.

BENDING STRESS



Procedure for Analysis

❖ Section Property.

- Determine moment of inertia I, of cross-sectional area about the neutral axis.
- Methods used are discussed in Textbook Appendix A.
- Refer to the course book's inside front cover for the values of I for several common shapes.

BENDING STRESS

Moment of Inertia of composite area

A composite area is made by adding or subtracting a series of "simple shaped areas" like

- **Rectangles:** $$I_{\substack{\text{centroidal}\\\text{x-axis}}} = \frac{bh^3}{12} \quad ; I_{\substack{\text{centroidal}\\\text{y-axis}}} = \frac{hb^3}{12}$$
- **Triangles:** $$I_{\substack{\text{centroidal}\\\text{x-axis}}} = \frac{bh^3}{36} \quad ; I_{\substack{\text{centroidal}\\\text{y-axis}}} = \frac{hb^3}{36}$$
- **Circles:** $$I_{\substack{\text{centroidal}\\\text{x-axis}}} = I_{\substack{\text{centroidal}\\\text{y-axis}}} = \frac{\pi r^4}{4} \text{ due to symmetry}$$

PARALLEL-AXIS THEOREM

$$I_y = \bar{I}_{y'} + Ad_x^{\,2} \qquad I_x = \bar{I}_{x'} + Ad_y^{\,2}$$

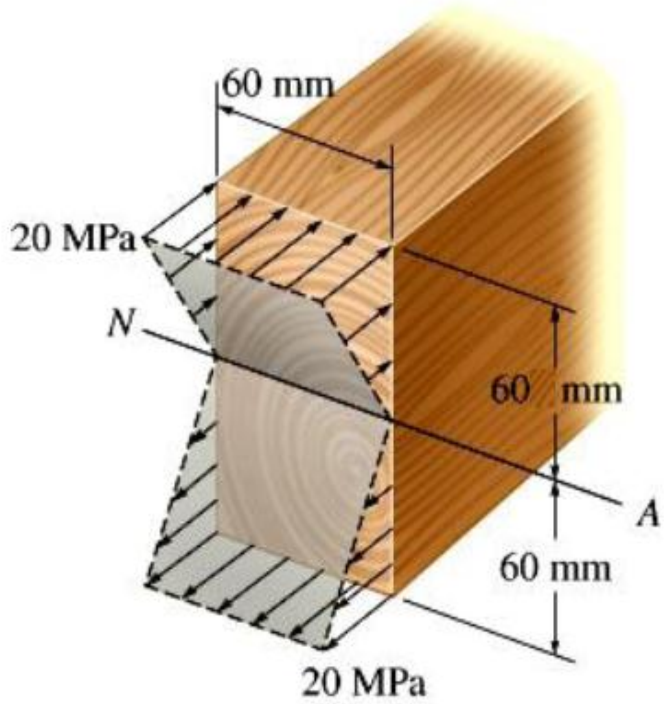
Procedure for Analysis

❖ Normal Stress.

- **Specify distance y, measured perpendicular to neutral axis to point where normal stress is to be determined.**
- **Apply equation $\sigma = -My/I$, or if maximum bending stress is needed, use $\sigma_{max} = Mc/I$.**
- **Ensure units are consistent when substituting values into the equations**

Ex1:-

A beam has a rectangular cross section and is subjected to the stress distribution shown in Fig. 6–27*a*. Determine the internal moment **M** at the section caused by the stress distribution (*a*) using the flexure formula, (*b*) by finding the resultant of the stress distribution using basic principles.



Solution

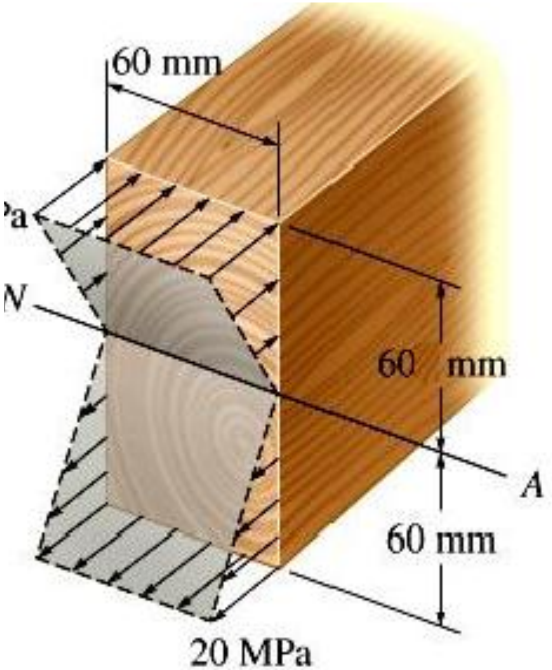
Part (a). The flexure formula is $\sigma_{max} = Mc/I$. From Fig. 6–27*a*, $c = 60$ mm and $\sigma_{max} = 20$ MPa. The neutral axis is defined as line *NA*, because the stress is zero along this line. Since the cross section has a rectangular shape, the moment of inertia for the area about *NA* is determined from the formula for a rectangle given on the inside front cover; i.e.,

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(60\text{ mm})(120\text{ mm})^3 = 864(10^4)\text{ mm}^4$$

Therefore,

$$\sigma_{max} = \frac{Mc}{I}; \qquad 20\text{ N/mm}^2 = \frac{M(60\text{ mm})}{864(10^4)\text{ mm}^4}$$

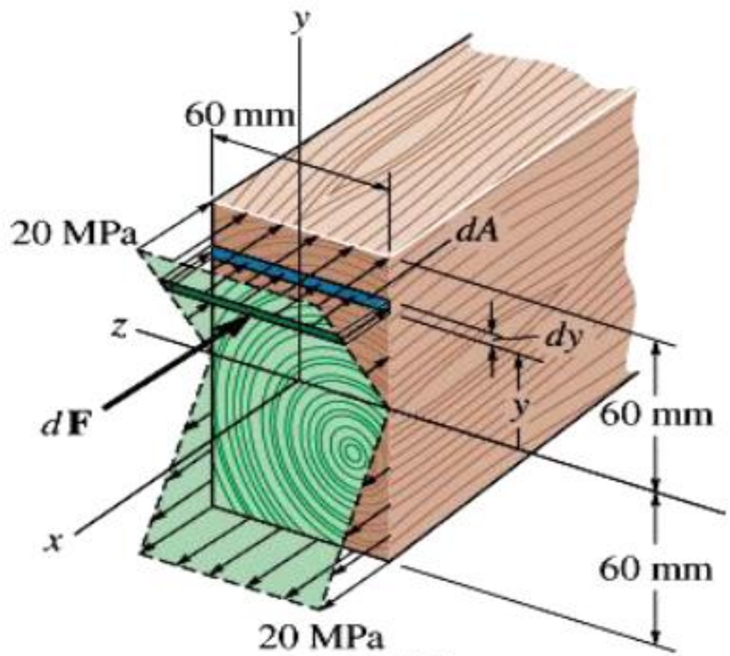
$$M = 288(10^4)\text{ N}\cdot\text{mm} = 2.88\text{ kN}\cdot\text{m}$$



(a)

Part (b). First we will show that the resultant force of the stress distribution is zero. As shown in Fig. 6–27*b*, the stress acting on the arbitrary element strip $dA = (60 \text{ mm})\, dy$, located y from the neutral axis, is

$$\sigma = \left(\frac{-y}{60 \text{ mm}}\right)(20 \text{ N/mm}^2)$$



(b)

BENDING STRESS

The force created by this stress is $dF = \sigma\, dA$, and thus, for the entire cross section,

$$F_R = \int_A \sigma\, dA = \int_{-60\text{ mm}}^{+60\text{ mm}} \left[\left(\frac{-y}{60\text{ mm}}\right)(20\text{ N/mm}^2)\right](60\text{ mm})\, dy$$

$$= (-10\text{ N/mm}^2)\, y^2 \Big|_{-60\text{ mm}}^{+60\text{ mm}} = 0$$

The resultant moment of the stress distribution about the neutral axis (z axis) must equal M. Since the magnitude of the moment of $d\mathbf{F}$ about this axis is $dM = y\, dF$, and $d\mathbf{M}$ is *always positive*, Fig. 6–27*b*, then for the entire area,

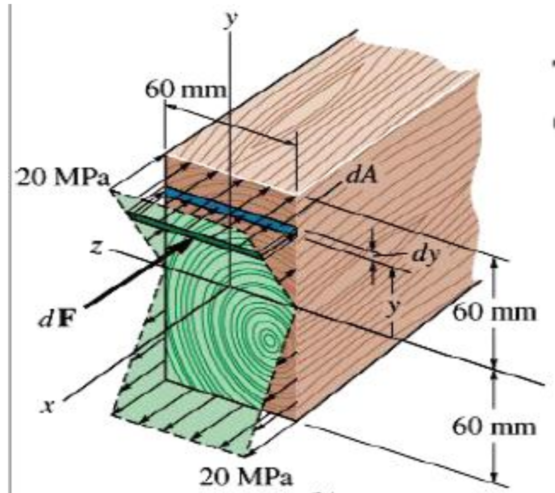
$$M = \int_A y\, dF = \int_{-60\text{ mm}}^{+60\text{ mm}} y\left[\left(\frac{y}{60\text{ mm}}\right)(20\text{ N/mm}^2)\right](60\text{ mm})\, dy$$

$$= \left(\frac{20}{3}\text{N/mm}^2\right) y^3 \Big|_{-60\text{ mm}}^{+60\text{ mm}}$$

$$= 288(10^4)\text{ N}\cdot\text{mm} = 2.88\text{ kN}\cdot\text{m}$$

dF

Ans.

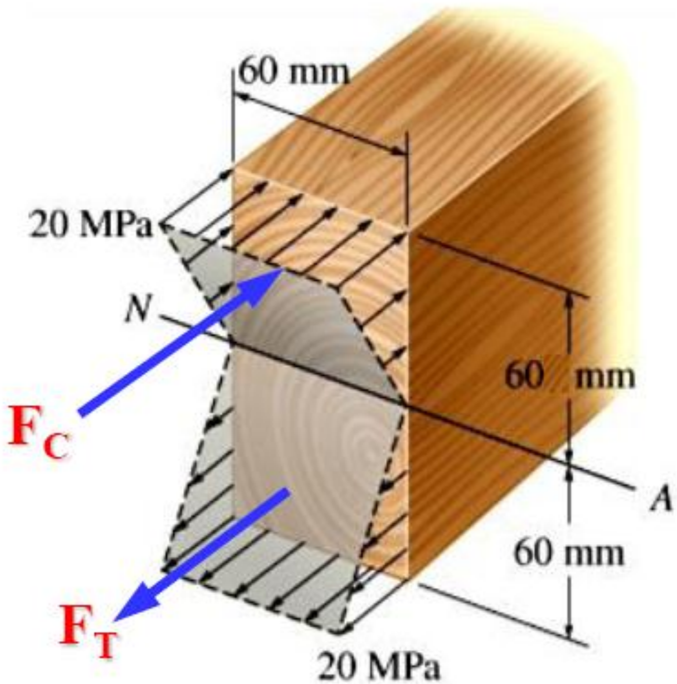


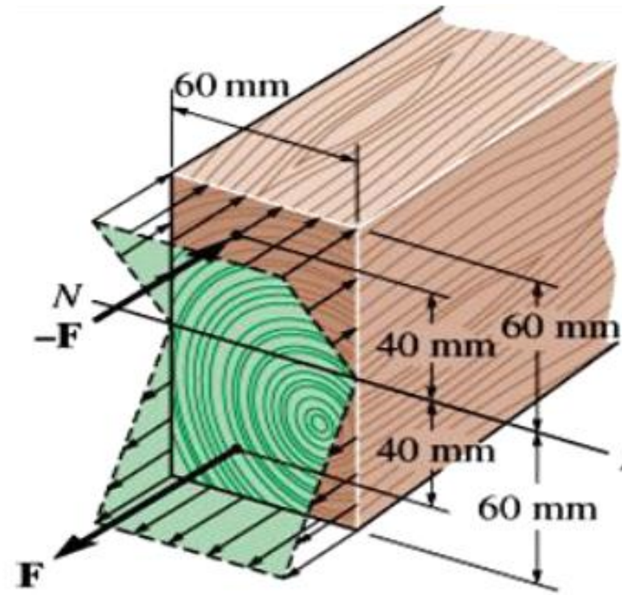
(b)

BENDING STRESS

The above result can *also* be determined without the need for integration. The resultant force for each of the two *triangular* stress distributions in Fig. 6–27*c* is graphically equivalent to the *volume* contained within each stress distribution. Thus, each volume is

$$F = \frac{1}{2}(60\text{ mm})(20\text{ N/mm}^2)(60\text{ mm}) = 36(10^3)\text{ N} = 36\text{ kN}$$



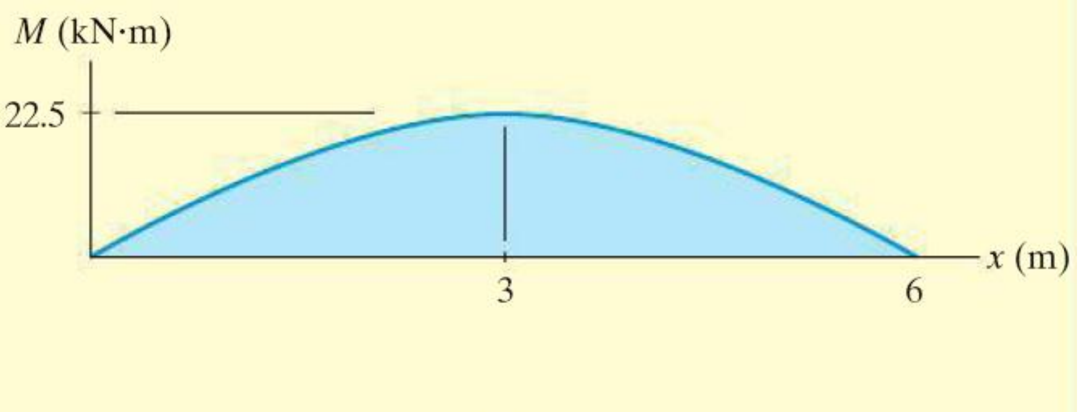


$$M = F_C \times 80 = 36000 \times 80 = 2880000\text{ N.mm}$$

BENDING STRESS

Maximum Internal Moment.

The maximum internal moment in the beam, $M = 22.5$ kN. m, occurs at the center.



(c)

Section Property.

By reasons of symmetry, the neutral axis passes through the centroid *C* at the mid height of the beam, Fig. 6–26 *b* . The area is subdivided into the three parts shown, and the moment of inertia of each part is calculated about the neutral axis using the parallel-axis theorem. (See Eq. A–5 of Appendix A .) Choosing to work in meters, we have

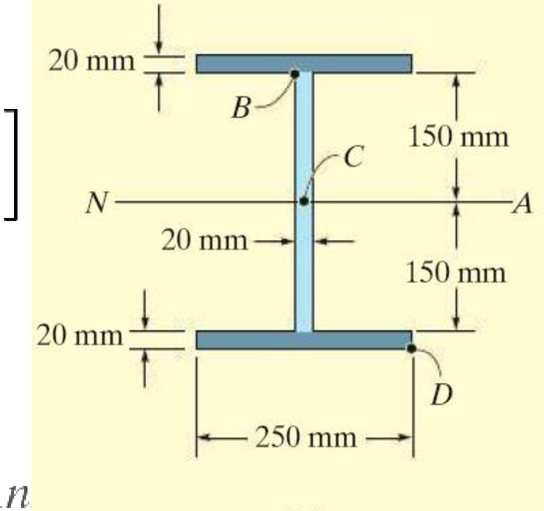
$$I = \Sigma(\bar{I} + A d^2)$$

$$= 2\left[\frac{1}{12}(0.25\text{ m})(0.020\text{ m})^3 + (0.25\text{ m})(0.020\text{ m})(0.160\text{ m})^2\right]$$

$$+ \left[\frac{1}{12}(0.020\text{ m})(0.300\text{ m})^3\right]$$

$$= 301.3(10^{-6})\text{ m}^4$$

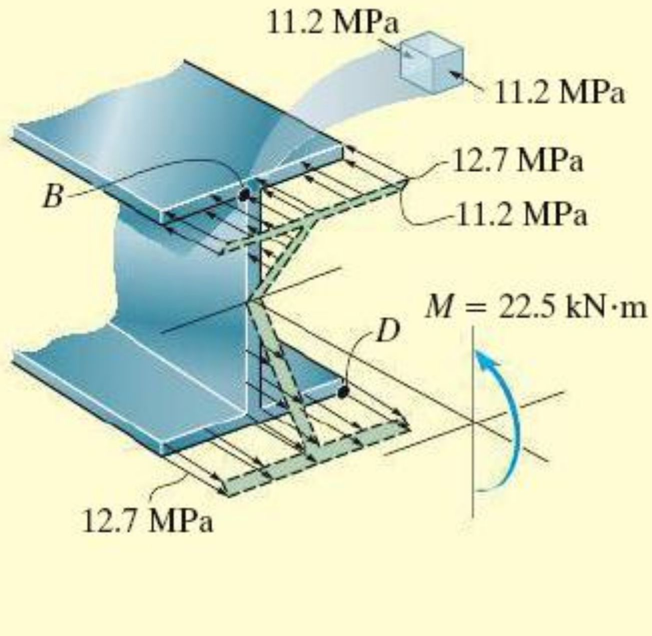
$$\sigma_{max} = \frac{Mc}{I}; \quad \sigma_{max} = \frac{22.5(10^3)\text{ N}\cdot\text{m}(0.170\text{ m})}{301.3(10^{-6})\text{ m}^4} = 12.7\text{ MPa} \quad An$$



(b)

BENDING STRESS

A three-dimensional view of the stress distribution is shown in Fig. 6–26*d*. Notice how the stress at points *B* and *D* on the cross section develops a force that contributes a moment about the neutral axis that has the same direction as **M**. Specifically, at point $B, y_B = 150$ mm, and so as shown in Fig. 6–26*d*,



(d)

Fig. 6–26

$$\sigma_B = -\frac{My_B}{I};$$

$$\sigma_B = -\frac{22.5(10^3)\ \text{N}\cdot\text{m}(0.150\ \text{m})}{301.3(10^{-6})\ \text{m}^4} =$$

$$= -11.2\ \text{MPa}$$